

General Certificate of Education  
June 2008  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Pure Core 1**

**MPC1**

Thursday 15 May 2008 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 The straight line  $L$  has equation  $y = 3x - 1$  and the curve  $C$  has equation

$$y = (x + 3)(x - 1)$$

- (a) Sketch on the same axes the line  $L$  and the curve  $C$ , showing the values of the intercepts on the  $x$ -axis and the  $y$ -axis. (5 marks)
- (b) Show that the  $x$ -coordinates of the points of intersection of  $L$  and  $C$  satisfy the equation  $x^2 - x - 2 = 0$ . (2 marks)
- (c) Hence find the coordinates of the points of intersection of  $L$  and  $C$ . (4 marks)

- 2 It is given that  $x = \sqrt{3}$  and  $y = \sqrt{12}$ .

Find, in the simplest form, the value of:

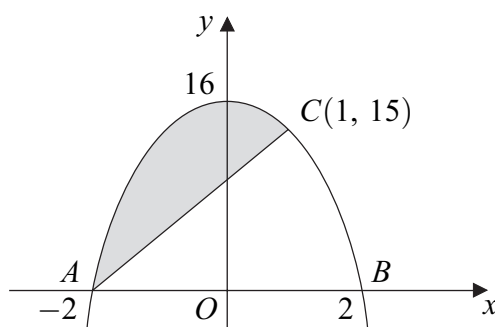
- (a)  $xy$ ; (1 mark)
- (b)  $\frac{y}{x}$ ; (2 marks)
- (c)  $(x + y)^2$ . (3 marks)

- 3 Two numbers,  $x$  and  $y$ , are such that  $3x + y = 9$ , where  $x \geq 0$  and  $y \geq 0$ .

It is given that  $V = xy^2$ .

- (a) Show that  $V = 81x - 54x^2 + 9x^3$ . (2 marks)
- (b) (i) Show that  $\frac{dV}{dx} = k(x^2 - 4x + 3)$ , and state the value of the integer  $k$ . (4 marks)
- (ii) Hence find the two values of  $x$  for which  $\frac{dV}{dx} = 0$ . (2 marks)
- (c) Find  $\frac{d^2V}{dx^2}$ . (2 marks)
- (d) (i) Find the value of  $\frac{d^2V}{dx^2}$  for each of the two values of  $x$  found in part (b)(ii). (1 mark)
- (ii) Hence determine the value of  $x$  for which  $V$  has a maximum value. (1 mark)
- (iii) Find the maximum value of  $V$ . (1 mark)

- 4 (a) Express  $x^2 - 3x + 4$  in the form  $(x - p)^2 + q$ , where  $p$  and  $q$  are rational numbers. (2 marks)
- (b) Hence write down the minimum value of the expression  $x^2 - 3x + 4$ . (1 mark)
- (c) Describe the geometrical transformation that maps the graph of  $y = x^2$  onto the graph of  $y = x^2 - 3x + 4$ . (3 marks)
- 5 The curve with equation  $y = 16 - x^4$  is sketched below.



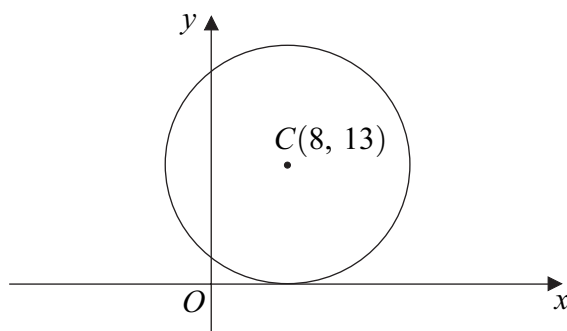
The points  $A(-2, 0)$ ,  $B(2, 0)$  and  $C(1, 15)$  lie on the curve.

- (a) Find an equation of the straight line  $AC$ . (3 marks)
- (b) (i) Find  $\int_{-2}^1 (16 - x^4) dx$ . (5 marks)
- (ii) Hence calculate the area of the shaded region bounded by the curve and the line  $AC$ . (3 marks)
- 6 The polynomial  $p(x)$  is given by  $p(x) = x^3 + x^2 - 8x - 12$ .
- (a) Use the Remainder Theorem to find the remainder when  $p(x)$  is divided by  $x - 1$ . (2 marks)
- (b) (i) Use the Factor Theorem to show that  $x + 2$  is a factor of  $p(x)$ . (2 marks)
- (ii) Express  $p(x)$  as the product of linear factors. (3 marks)
- (c) (i) The curve with equation  $y = x^3 + x^2 - 8x - 12$  passes through the point  $(0, k)$ . State the value of  $k$ . (1 mark)
- (ii) Sketch the graph of  $y = x^3 + x^2 - 8x - 12$ , indicating the values of  $x$  where the curve touches or crosses the  $x$ -axis. (3 marks)

**Turn over for the next question**

**Turn over ►**

7 The circle  $S$  has centre  $C(8, 13)$  and touches the  $x$ -axis, as shown in the diagram.



(a) Write down an equation for  $S$ , giving your answer in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (2 \text{ marks})$$

(b) The point  $P$  with coordinates  $(3, 1)$  lies on the circle.

(i) Find the gradient of the straight line passing through  $P$  and  $C$ . (1 mark)

(ii) Hence find an equation of the tangent to the circle  $S$  at the point  $P$ , giving your answer in the form  $ax + by = c$ , where  $a$ ,  $b$  and  $c$  are integers. (4 marks)

(iii) The point  $Q$  also lies on the circle  $S$ , and the length of  $PQ$  is 10. Calculate the shortest distance from  $C$  to the chord  $PQ$ . (3 marks)

8 The quadratic equation  $(k + 1)x^2 + 4kx + 9 = 0$  has real roots.

(a) Show that  $4k^2 - 9k - 9 \geq 0$ . (3 marks)

(b) Hence find the possible values of  $k$ . (4 marks)

**END OF QUESTIONS**